Hypergraph $T$-Coloring for
Automatic Frequency Planning problem
in Wireless LAN

Cannes – 16 September 2008

Alexandre Gondran - Oumaya Baala
Hakim Mabed - Alexandre Caminada
Agenda

1. Channel Assignment in Wireless LAN based on SINR
2. Necessary condition: Graph $T$-coloring Problem
3. Quasi equivalent condition: Hypergraph $T$-coloring Problem
4. Results
5. Conclusions/Perspectives
Channel Assignment in WLAN

Limit the interferences which degrade Quality of Service network by limiting its capacity.

Not possible to avoid interferences to spread as well as possible interferences over the whole area

⇒ SINR computation : long time

Signal-to-Interference-plus-Noise-Ratio

SINR constraints
Channel Assignment in WLAN: 
SINR constraints

- Frequency channel used by Access Point $i$: $f_i$
- SINR threshold necessary to satisfy user $k$: $s_k$

$\forall k$, $\text{SINR}_{k} \geq s_k$

$\text{SINR} \geq 15 \text{ dB}$
$\text{SINR} \geq 20 \text{ dB}$
$\text{SINR} \geq 25 \text{ dB}$

Determine $f_i$
Channel Assignment in WLAN:
SINR constraints

Determine \( f_i \)
Satisfied: \( \forall k, \ \text{SINR}_k(f_i) \geq s_k \)

\[
\text{SINR}_k = \frac{P_{i_k}}{\sum_{i \neq i_k} p_{ik} \times \gamma(|f_{i_k} - f_i|) + N}
\]
SINR constraints

\[
SINR_k = \frac{p_{ik}}{\sum_{i \neq i_k} p_{ik} \times \gamma(\lfloor f_{ik} - f_i \rfloor) + N} \geq s_k
\]

Graph

\[|f_j - f_i| \geq t_{ij}\]

\[
t_{ij} = \max_k \left\{ \gamma^{-1} \left( \frac{p_{ik}}{s_k} - N \right), j = i_k ; \gamma^{-1} \left( \frac{p_{jk}}{p_{ik}} - N \right), i = j_k \right\}
\]
Necessary condition:
Graph $T$-coloring problem

Example 1

$SINR \geq 15dB$
Necessary condition:

Graph $T$-coloring problem

$SINR \geq 15 \iff \left\{ \begin{align*}
|f_1 - f_2| &\geq 2 \\
|f_1 - f_3| &\geq 2
\end{align*} \right\}$
SINR constraints

\[ \text{SINR}_k = \frac{p_{i,k}}{\sum_{i \neq i_k} p_{i,k} \times \gamma(|f_{i,k} - f_i|) + N} \geq s_k \]

Theorem:
Yes, if \( \forall k, \quad \text{SINR}_k := \frac{p_{i,k}}{\sum_{i \neq i_k} p_{i,k} \times \gamma(t_{i,k}) + N} \geq s_k \)
Quasi equivalent condition:
Hypergraph $T$-coloring problem

Example 2

- 63 dBm
- 73 dBm
- 60 dBm
- 72 dBm
Quasi equivalent condition:

Hypergraph $T$-coloring problem

$SINR \geq 15 \implies \begin{cases} |f_1 - f_2| \geq 2 \\ |f_1 - f_3| \geq 3 \end{cases}$
Quasi equivalent condition:

Hypergraph $T$-coloring problem

\[
SINR \geq 15 \iff \begin{cases} 
|f_1 - f_2| \geq 2 \\
|f_1 - f_3| \geq 3 
\end{cases}
\]

no equivalence

It’s necessary to add a new constraint

linear n-ary constraints: \[|f_1 - f_2| + |f_1 - f_3| \geq 6\]

\[
SINR \geq 15 \iff \begin{cases} 
|f_1 - f_2| \geq 2 \\
|f_1 - f_3| \geq 3 \\
|f_1 - f_2| + |f_1 - f_3| \geq 6
\end{cases}
\]
SINR constraints

\[ \text{SINR}_k = \frac{p_{ik}}{\sum_{i \neq i_k} p_i \times \gamma(|f_{ik} - f_i|) + N} \geq s_k \]

Graph \( T \)-coloring
\[ |f_j - f_i| \geq t_{ij} \]

Hypergraph \( T \)-coloring
\[ \sum_{i \neq i_k} \alpha_{ik} |f_i - f_{ik}| \geq \alpha_{ii_k} \]

\[ \forall k, \forall i \neq i_k, \quad \alpha_{ik} = \frac{1}{\min_t} \left( p_{jk} \gamma(t_{jk} + t) + \sum_{i \neq j} p_{ik} \gamma(t_{ik}) \leq \frac{p_{ik}}{s_k} - N \right) \]
\[ \text{SINR constraints} \]
\[ SINR_k = \frac{p_{ik}}{\sum_{i \neq i_k} p_{ik} \times \gamma(|f_{ik} - f_i|) + N} \geq s_k \]

\[ \text{equivalence?} \]

**Theorem:** Yes, if \( \forall k, \forall i \neq i_k, \quad \alpha_{ik} = 1 \)
## Results

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Conclusions / Perspectives

• Automatic method to build a good $T$ matrix for graph $T$-coloring problem.

• Define a new problem: hypergraph $T$-coloring problem.

• Results equivalents for a better computation time.

• Benchmarks soon in Internet: http://alexandre.gondran.free.fr

• Integrate this approach into a global WLAN planning process.
Remark

\[ SINR_k = \frac{p_{ik}}{\sum_{i \neq i_k} p_{ik} \times \gamma(|f_{ik} - f_i|) + N} \geq s_k \]

In real problem, threshold \( s_k \) are unknown
Only throughput demand per user (kilobit/s) are known

Graph
\[ |f_j - f_i| \geq t_{ij} \]

Hypergraph
\[ \sum_{i \neq i_k} \alpha_{ik} |f_i - f_{ik}| \geq \alpha_{ii_k} \]
SINR constraints

\[ SINR_k = \frac{p_{i,k}}{\sum_{i \neq i_k} p_{i,k} \times \gamma(|f_{i,k} - f_i|) + N} \geq s_k \]

In real problem, threshold \( s_k \) are unknown
Only throughput demand per user (kilobit/s) are known

Capacity constraints
\[ \text{Capacity}_{AP} \geq \text{Demand} \]

Graph \( T \)-coloring
\[ |f_j - f_i| \geq t_{ij} \]

Hypergraph \( T \)-coloring
\[ \sum_{i \neq i_k} \alpha_{ik} |f_i - f_{i,k}| \geq \alpha_{ii_k} \]
We define a new algorithm which determine dynamically the best threshold $s_k$ to transform the problem into graph and hypergraph $T$-coloring problem.

SINR constraints

$$SINR_k = \frac{p_{ik}}{\sum_{i\neq i_k} p_{ik} \times \gamma(|f_{ik} - f_i|) + N} \geq s_k$$

Capacity constraints

$Capacity_{AP} \geq Demand$

Graph $T$-coloring

$|f_j - f_i| \geq t_{ij}$

Hypergraph $T$-coloring

$\sum_{i\neq i_k} \alpha_{ik} |f_i - f_{ik}| \geq \alpha_{ii_k}$
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