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Hypergraph *T*-Coloring for Automatic Frequency Planning problem in Wireless LAN

Cannes – 16 September 2008

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Agenda

1. Channel Assignment in Wireless LAN based on SINR
2. Necessary condition: Graph T-coloring Problem
3. Quasi equivalent condition: Hypergraph T-coloring Problem
4. Results
5. Conclusions/Perspectives

Channel Assignment in WLAN

Limit the interferences
which degrade Quality of Service network
by limiting its capacity.

Not possible to avoid interferences
→ to spread as well as possible interferences
over the whole area

➔ SINR computation : long time

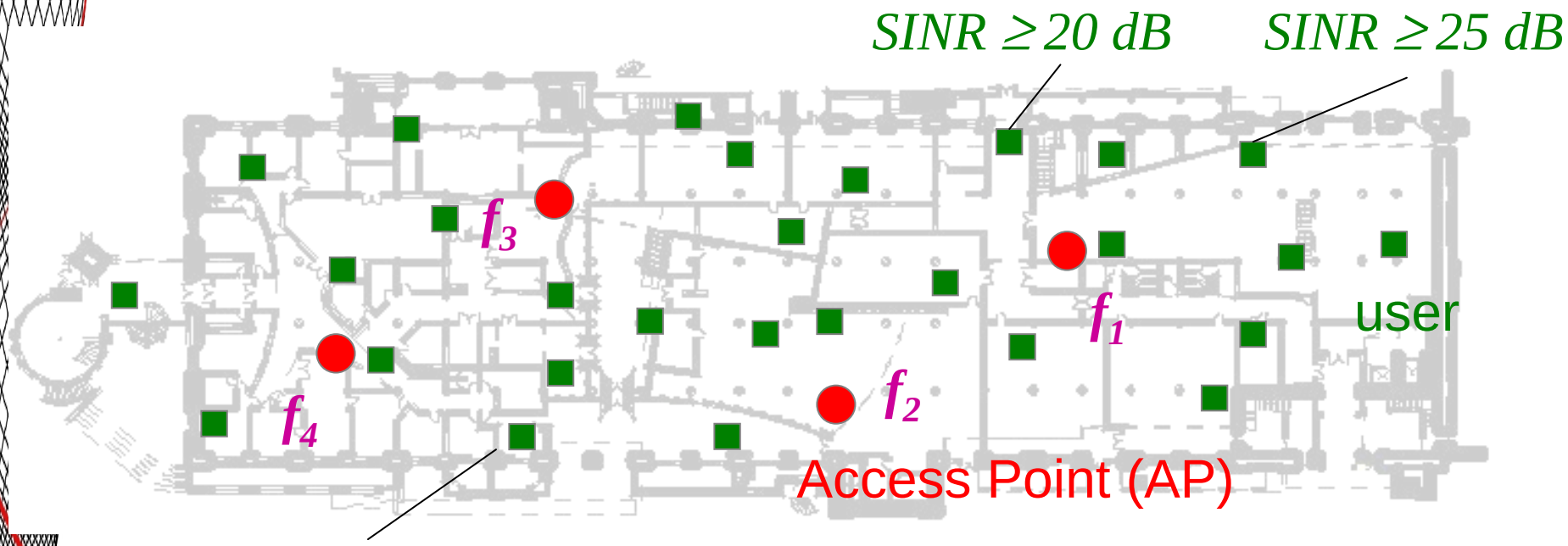
Signal-to-Interference-plus-Noise-Ratio

SINR constraints

Channel Assignment in WLAN: SINR constraints

f_i frequency channel used by Access Point i

s_k SINR threshold necessary to satisfied user k



Determine f_i

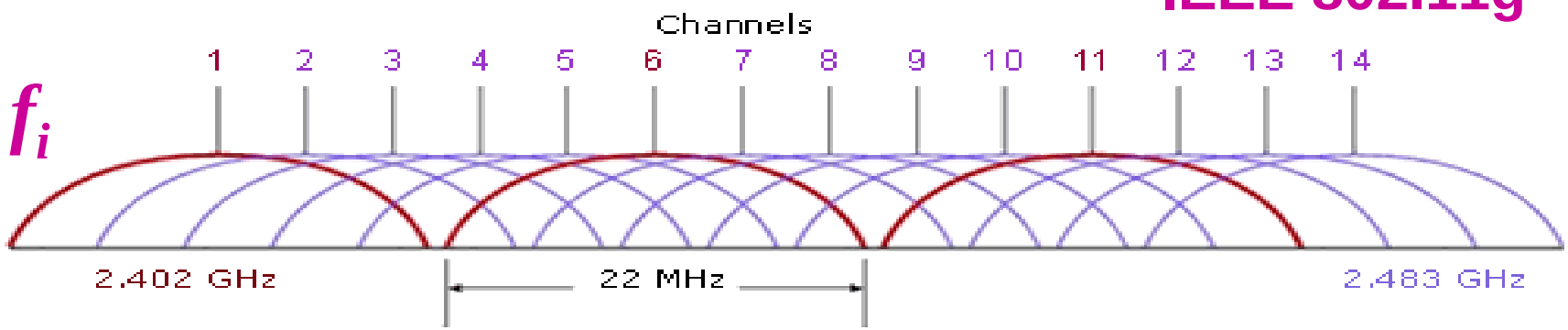
Satisfied : $\forall k,\ SINR_k \geq s_k$

Channel Assignment in WLAN: SINR constraints

Determine f_i

Satisfied : $\forall k,\ SINR_k(f_i) \geq s_k$

IEEE 802.11g



$$SINR_k = \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(\left| f_{i_k} - f_i \right|\right) + N}$$

SINR constraints

$$SINR_k = \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(\left| f_{i_k} - f_i \right|\right) + N} \geq s_k$$

Graph

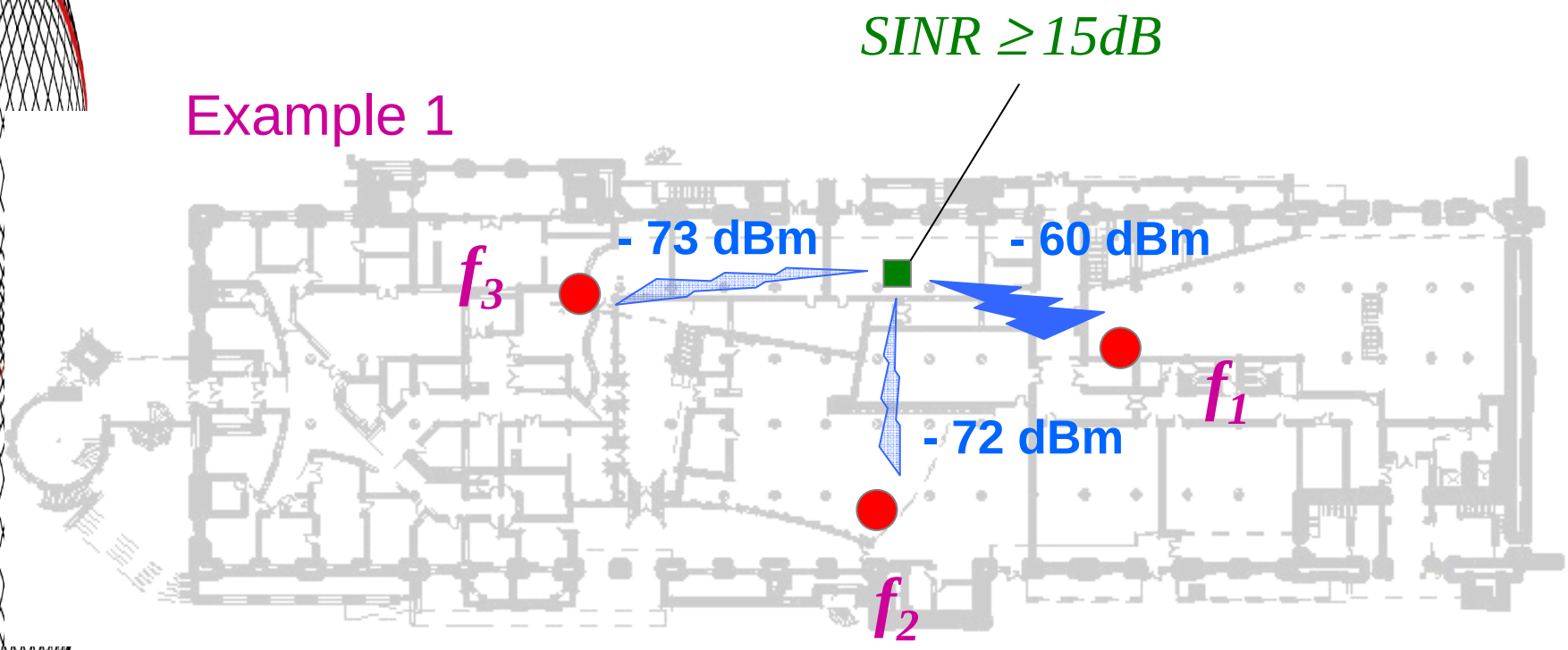
T-coloring

$$| f_j - f_i | \geq t_{ij}$$

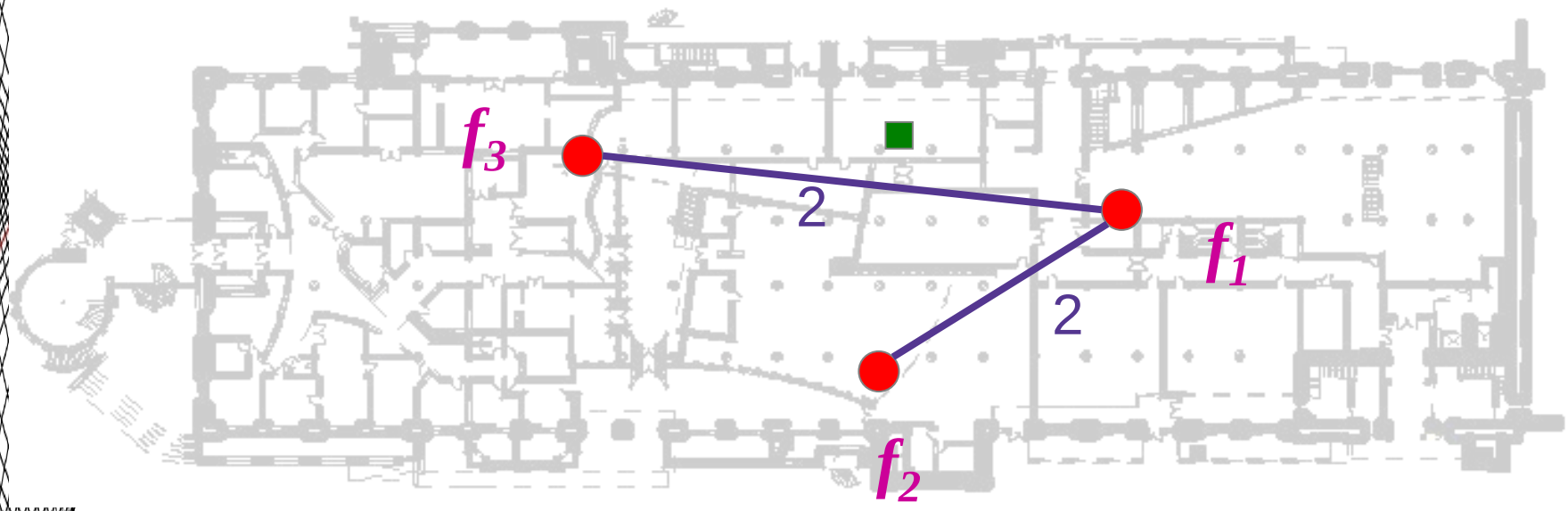
$$t_{ij} = \max_k \left(\gamma^{-1}\left(\frac{p_{i_k k} / s_k - N}{p_{ik}} \right), j = i_k ; \gamma^{-1}\left(\frac{p_{i_k k} / s_k - N}{p_{jk}} \right), i = j_k \right)$$

Necessary condition:
Graph *T*-coloring problem

Example 1



Necessary condition:
Graph *T*-coloring problem



$$SINR \geq 15 \Leftrightarrow \begin{cases} |f_1 - f_2| \geq 2 \\ |f_1 - f_3| \geq 2 \end{cases}$$

SINR constraints

$$SINR_k = \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(\left| f_{i_k} - f_i \right|\right) + N} \geq s_k$$

equivalence ?

Graph

T-coloring

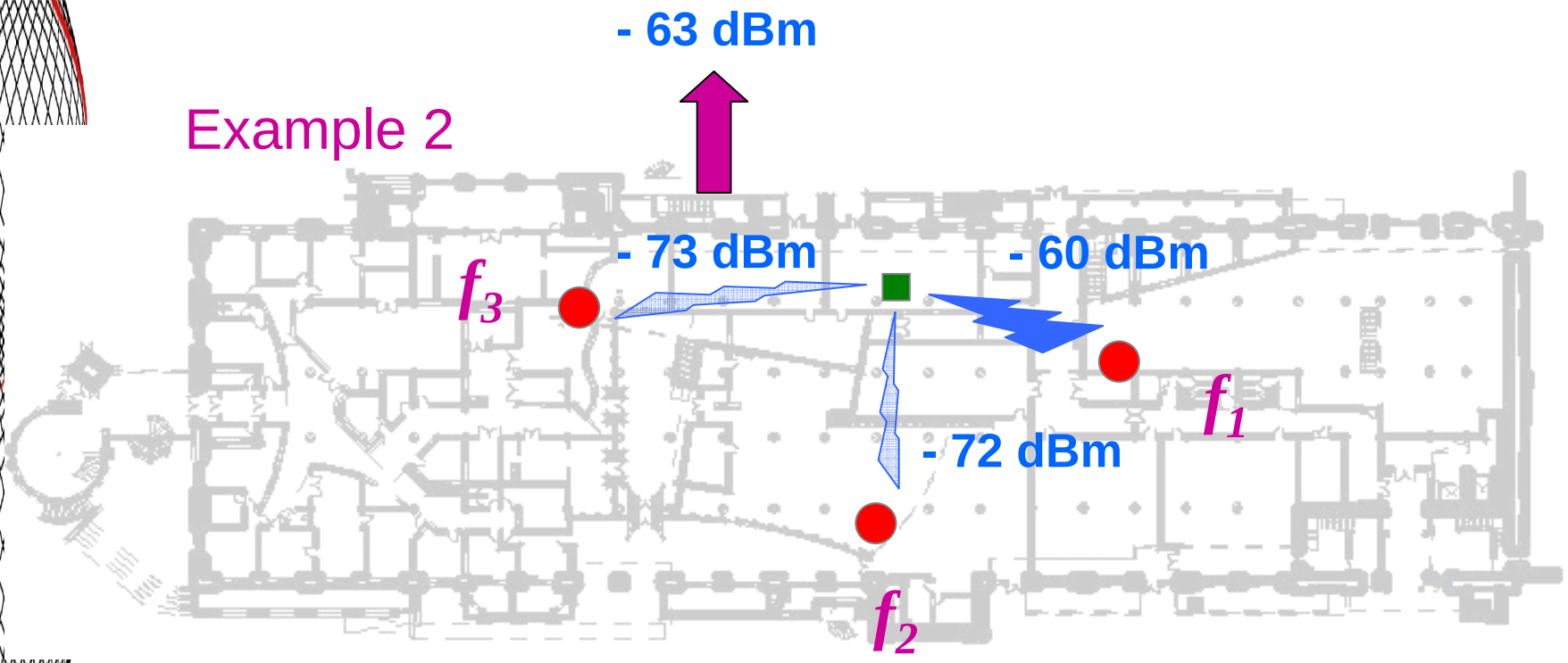
$$| f_j - f_i | \geq t_{ij}$$

Theorem:

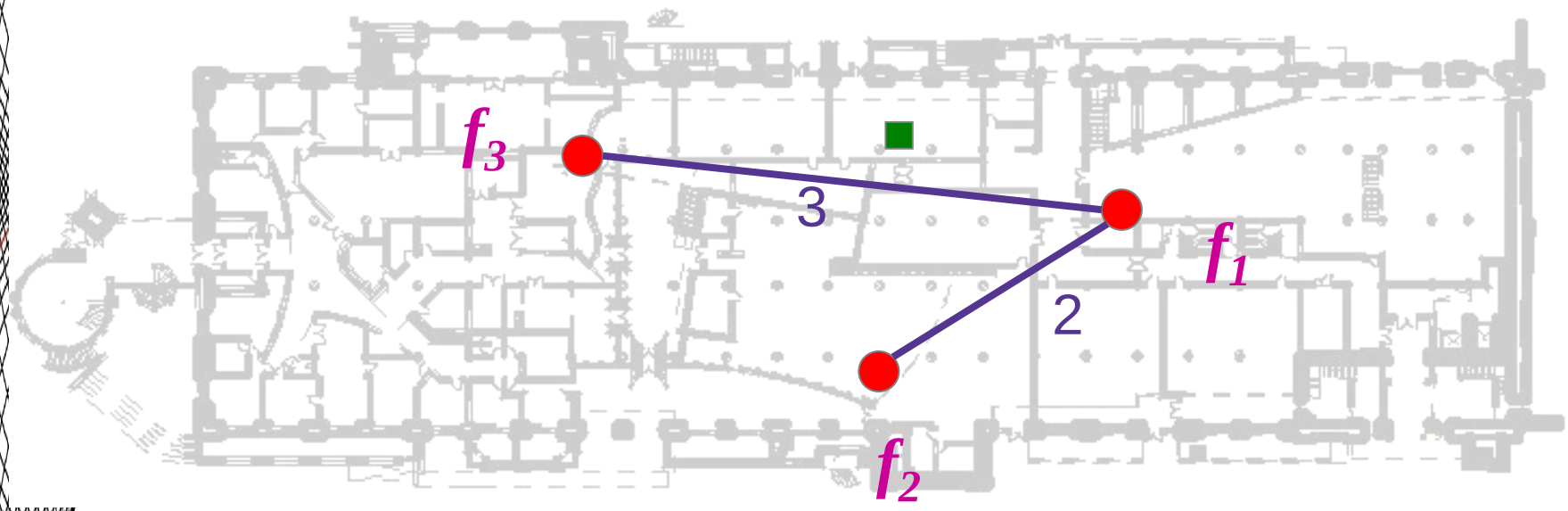
Yes, if $\forall k, \quad \underline{SINR}_k := \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(t_{ii_k}\right) + N} \geq s_k$

Quasi equivalent condition: Hypergraph *T*-coloring problem

Example 2



Quasi equivalent condition:
Hypergraph *T*-coloring problem



$$SINR \geq 15 \Rightarrow \begin{cases} |f_1 - f_2| \geq 2 \\ |f_1 - f_3| \geq 3 \end{cases}$$

Quasi equivalent condition:
Hypergraph T-coloring problem

$$SINR \geq 15 \;\not\Leftrightarrow\; \begin{cases} |f_1 - f_2| \geq 2 \\ |f_1 - f_3| \geq 3 \end{cases}$$

no equivalence

It's necessary to add a new constraint

➔ linear n-ary constraints : $|f_1 - f_2| + |f_1 - f_3| \geq 6$

$$SINR \geq 15 \;\Leftrightarrow\; \begin{cases} |f_1 - f_2| \geq 2 \\ |f_1 - f_3| \geq 3 \\ |f_1 - f_2| + |f_1 - f_3| \geq 6 \end{cases}$$

SINR constraints

$$SINR_k = \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(\left| f_{i_k} - f_i \right|\right) + N} \geq s_k$$

Graph T-coloring

$$| f_j - f_i | \geq t_{ij}$$

Hypergraph T-coloring

$$\sum_{i \neq i_k} \alpha_{ik} \, | f_i - f_{i_k} | \geq \alpha_{ii_k}$$

$$\forall k, \forall i \neq i_k, \quad \alpha_{ik} = 1 \Big/ \min_t \left(p_{jk} \gamma(t_{ji_k} + t) + \sum_{i \neq j} p_{ik} \gamma(t_{ii_k}) \leq \frac{p_{i_k k}}{s_k} - N \right)$$

SINR constraints

$$SINR_k = \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(\left| f_{i_k} - f_i \right|\right) + N} \geq s_k$$

equivalence ?

Graph
T-coloring

$$| f_j - f_i | \geq t_{ij}$$

Hypergraph
T-coloring

$$\sum_{i \neq i_k} \alpha_{ik} \, | f_i - f_{i_k} | \geq \alpha_{ii_k}$$

Theorem:

Yes, if $\forall k, \forall i \neq i_k, \quad \alpha_{ik} = 1$

Results

	Tests	Problem 0		Problem 1		Problem 2	
		SINR constraints		Graph T-coloring		Hypergraph T-coloring	
	ID-$n/m/SINR/it.$	time (s)	f_0	time (s)	f_0	time (s)	f_0
9 AP	1-9/3024/4/100	10	0	0.5	0	2.5	0
	2-9/3024/16/1000	187	1.8	9	2.1	59	3.2
	3-9/3024/22/1000	191	20.6	10	21.2	54	21.2
	4-9/3024/24/1000	187	29.2	10	29.2	56	29.2
	5-9/3024/30/1000	197	85.4	10	96.7	59	96.7
15 AP	6-15/3024/4/100	35	0	2	0.5	13	0
	7-15/3024/16/1000	689	20	37.7	24.2	146	21.8
	8-15/3024/22/1000	652	70.9	39	79.8	136	79.8
	9-15/3024/24/1000	624	89.8	41	111.6	149	111.6
	10-15/3024/30/1000	616	137	43	160	136	158.4
30 AP	11-30/7728/4/100	835	0	37	0.6	108	0.4
	12-30/7728/16/200	2182	35.6	95.6	36.1	393	35.1
	13-30/7728/22/100	1164	78.5	45	80.4	180	73.5
	14-30/7728/24/100	3416	101.3	157	103	668	104.1
	15-30/7728/30/100	1085	178.5	45	186.5	188	186.5
40 AP	16-40/7728/4/100	1392	0	85.7	1.3	192	0
	17-40/7728/16/100	2028	33	146	33.6	421	35.6
	18-40/7728/22/100	2231	83.7	137	86.2	428	83.3
	19-40/7728/24/100	2090	107	142	112.3	428	112.3
	20-40/7728/30/100	2704	201.4	149	205.6	415	201.1

Conclusions / Perspectives

- Automatic method to build a good *T* matrix for graph *T*-coloring problem.
- Define a new problem:
  hypergraph *T*-coloring problem.
- Results equivalents for a better computation time.

- Benchmarks soon in Internet:
  http://alexandre.gondran.free.fr
- Integrate this approach into a global WLAN planning process.

Remark

SINR constraints

$$SINR_k = \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(\left| f_{i_k} - f_i \right|\right) + N} \geq s_k$$

In real problem, threshold s_k are unknown
Only throughput demand per user
(kilobit/s) are known

Graph
T-coloring

$$| f_j - f_i | \geq t_{ij}$$

Hypergraph
T-coloring

$$\sum_{i \neq i_k} \alpha_{ik} | f_i - f_{i_k} | \geq \alpha_{ii_k}$$

SINR constraints

$$SINR_k = \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(\left| f_{i_k} - f_i \right|\right) + N} \geq s_k$$

In real problem, threshold s_k are unknown
Only throughput demand per user (kilobit/s) are known

Capacity constraints

$$Capacity_{AP} \geq Demand$$

Graph T-coloring

$$| f_j - f_i | \geq t_{ij}$$

Hypergraph T-coloring

$$\sum_{i \neq i_k} \alpha_{ik} \mid f_i - f_{i_k} \mid \geq \alpha_{ii_k}$$

SINR constraints

$$SINR_k = \frac{p_{i_k k}}{\sum_{i \neq i_k} p_{ik} \times \gamma\left(\left| f_{i_k} - f_i \right|\right) + N} \geq s_k$$

We define a new algorithm which determine dynamically the best threshold s_k to transform the problem into graph and hypergraph T-coloring problem

Capacity constraints

$$Capacity_{AP} \geq Demand$$

Graph T-coloring

$$| f_j - f_i | \geq t_{ij}$$

Hypergraph T-coloring

$$\sum_{i \neq i_k} \alpha_{ik} \, | f_i - f_{i_k} | \geq \alpha_{ii_k}$$

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