Hypergraph T-Coloring for Automatic Frequency Planning problem in Wireless LAN

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***Abstract*—Frequency assignment is one of the main issues in radio networks planning. The multiple interferences are seldom taken into account in literature. There is not a framework with their modeling. A hypergraph modeling of the network gives a more realistic representation of this phenomenon.**

We generalize the *T*-coloring problem for graphs to hypergraphs. We apply this new modeling to IEEE 802.11b/g wireless networks and study its interest.

I. Introduction

Frequency management is one of the main issues in radio networks planning. It aims to limit the interferences which degrade Quality of Service (QoS) network by limiting its capacity. However, it is often not possible to avoid interferences, the goal is thus to spread as well as possible interferences over the whole area.

Frequency assignment problems are often modeled by k-coloring problems or T-coloring for graph [1][2][3]. However, the concept of graph is restrictive because it corresponds to binary relations on sets. But interferences are often multiple; they come from several transmitters simultaneously thus their conjunctions penalize the network. A network modeling using a hypergraph [4] allows a more realistic representation.

The paper is organized as follows. First, we remind the frequency assignment problem for IEEE 802.11b/g *Wireless Local Area Networks* (*WLAN*) and the calculation of the Signal to Interference plus Noise Ratio (*SINR*). Then, we introduce a formalism based on hypergraphs denoted *Problem 0*. We transform it into a graph T-coloring problem denoted *Problem 1*. Since this problem is under constrained, we introduce the hypergraph T-coloring problem denoted *Problem 2* in order to correct this simplification. Finally a frequency assignment algorithm is proposed to compare the performance of these two T-coloring approaches.

II. Frequency Channel Assignment

The objective is to allocate one of the available frequencies to each Access Point (AP) configuration in order to minimize interferences. The available frequency set depends on the standard (IEEE 802.11 a, b or g) and also on specific restriction on spectrum usage in each country and environment. This problem is called AFP problem for **Automatic Frequency Planning** and becomes very famous for designing GSM/GPRS/EDGE cellular network [5][6][7][8]. Early studies related to interferences management in IEEE 802.11 context do not treat directly the channel assignment. Instead, they integrate various constraints to AP placement problem. For example prohibiting the selection of two close sites [9][10] or minimizing the overlapping area between cells [11][12][13] or selecting BSS according to its geometrical shape [14] as in cellular [15]. Another approach is to estimate the capacity of channel frequency reuse in WLAN system [16] or in cellular system [17], using hypergraph model. More sophisticated approach is to evaluate the deviation between interfering transmitter [18]. Those works introduce more complete AFP problem in IEEE 802.11 [19][20][21][3]. However [22][23][24] use only three non-overlapping channels. Complete AFP problem based on SINR total calculation is done in [25][26].

A. Problem data

Let us introduce some notations that help defining the problem. To characterize the users mobility in the network, service zones are defined. Each zone is characterized by a number of users and a level of Quality of Service (QoS). To each QoS level corresponds a *SINR* threshold. Each service zone is decomposed into Service Points (SP) corresponding to one square meter.

- I is the set of AP, $|I| = n$.
- J is the set of SP, $|J| = m$.
- u_j is the number of user characterizing the SP_j.
- s_j is the *SINR* threshold necessary to satisfy the QoS of SP_j. In the next section, we remind the definition of the *SINR*.
- p_{ij} is the power of the received signal by the SP_j from the AP_i, called Received Signal Strength (*RSS*). If $p_{ij} < -110dBm$, the SP_j does not perceive the AP_i. We denote AP_{i^*}, the AP from which the SP_j perceives the highest RSS also called the AP server, so $p_{i^*j} = max(p_{ij}, i \in I)$. Others signals are jammers. The set of SP communicating with the same AP_i is called the Basic Set Service (BSS_i).
- IEEE 802.11b/g has 14 overlapping frequency channels but only 13 channels are available in France. Owing to the standard definition, only 3 channels are not overlapping. We define $\gamma(.)$ the protection factor corresponding to the attenuation coefficient between two channels. It is a function of Δf, the channel distance between the carrier signal and the interfering signal. γ decreases when Δf increases: if $\Delta f = 0$, $\gamma = 1$ and if $\Delta f \geq 5$, $\gamma = 0$. All intermediate values depend on the receiver equipment features.

The problem variables are the frequency channels necessary to assign to each AP.

- $x_i \in D$ is the frequency channel number used by the AP_i with $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\}$ the set of available frequency channels for IEEE 802.11b/g standard. We define $X = \{x_1, x_2, ..., x_n\} \in D^n$ as a solution of the problem.

The quality of a radio link is given by the *SINR*.

B. Signal to Interference plus Noise Ratio - SINR

To simplify modeling, only the downlink interferences (from AP to user) are considered. We can easily generalize this definition to the uplink interferences (from user to AP). To measure the interferences on the level of SP_j, we calculate the *SINR* defined as:

$$SINR_j = \frac{p_{i^*j}}{\sum_{i \neq i^*} p_{ij}\gamma(|x_{i^*} - x_i|) + N} \tag{1}$$

with N the thermal noise strength. Its value is around $-100dBm$ in surrounding air (25^oC). The formula is valid for power values expressed in *Watt* unit. Higher is the *SINR*, better is the radio link. Thus, it is possible to code and modulate the signal more sophisticatedly, which allows higher throughput.

C. Objective - Problem 0

The objective of this AFP problem is to allocate a frequency channel to each antenna in order to satisfy the *QoS constraints* :

$$\forall j \in J,\ SINR_j \geq s_j \tag{2}$$

Taking into account *SINR* definition (1), the m QoS constraints (2) become :

$$\forall j \in J, \sum_{i \neq i^*} p_{ij}\gamma(|x_{i^*} - x_i|) \leq \frac{p_{i^*j}}{s_j} - N \tag{3}$$

For each SP_j, the sum of the interferences must be lower than a threshold $\tau_j = \frac{p_{i^*j}}{s_j} - N$. Let $\beta_{ij}^k = p_{ij}\gamma(k)$ be the contribution of jammer signals to SP_j if $|x_{i^*} - x_i| = k$ where $k \in \{0, 1, 2, 3, 4\}$. Considering this notation equation (3) is equivalent to :

$$\forall j \in J, \sum_{i \neq i^*} \beta_{ij}^{|x_{i^*} - x_i|} \leq \tau_j \tag{4}$$

The problem can be expressed as a Constraint Satisfaction Problem (CSP) ; $CSP_0 \ : \ (X, D^n, C0)$. The aim is to determine $X \in D^n$ satisfying the set of the constraints $C0 = \{C0_1, C0_2, ..., C0_m\}$ such as $C0_j \ : \sum_{i \neq i^*} \beta_{ij}^{|x_{i^*} - x_i|} \leq \tau_j$. While dealing with real problems, it is necessary to find a solution even if it does not satisfy all the constraints (2). Relaxi,g these constraints results in an optimization problem. As all constraints violations do not often have the same importance we allocate a penalty to each nonsatisfied constraint. In our case, the penalty is equal to u_j, the number of users corresponding to the constraint $C0_j$. The objective is to determine $X \in D^n$ which minimizes the fonction :

$$f_0(X) = \sum_j u_j \delta_{C0_j} \tag{5}$$

such as $\delta_{C0_j} = \begin{cases} 1 & \text{if constraint } C0_j \text{ is unsatisfied} \\ 0 & \text{otherwise} \end{cases}$.

The Problem 0 can be represented by a hypergraph $H = (V, \xi)$ with $V = \{AP_1, AP_2, ..., AP_n\}$ the set of vertices and $\xi = \{E_1, E_2, ..., E_m\}$ a family of V parts. A hyperedge E_j corresponds to each SP_j; The AP server of SP_j and the AP jammers of SP_j belong to E_j. To each hyperedge we associate to E_j the constraint $C0_j$. Figure 1 shows a graphic representation of a hyperedge E_j where AP_1 is the AP server of SP_j, AP_2 and AP_3 are jammers.

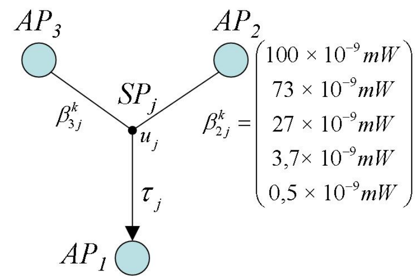

Fig. 1. Graphic representation of a hyperedge E_j associated to SP_j. The constraint associated to this hyperege is :$\beta_{2j}^{|x_1 - x_2|} + \beta_{3j}^{|x_1 - x_3|} \leq \tau_j$. The associated penalty is u_j. In this example, the hyperedge is represented by the node SP_j (•) connected to AP_1, AP_2 and AP_3.

III. HYPERGRAPH T-COLORING

T-coloring problems for graphs appeared in the eighties [1] to represent relations of deviation to be respected **between two variables**. Those are NP-complete problems. Many applications can be modeled as T-coloring problem like frequency assignment, setting in phase of traffic light, traffic management, tasks scheduling [1][27][28][29]...

The problem consists to affect one color (or several colors in the case of Set T-coloring [30]) to the graph vertices by respecting colors deviations between two vertices. For AFP problem in WLAN, we show that it is more realistic to represent relations of deviation **between more than two variables**. Then, we introduce more formally two problems of hypergraphs T-coloring.

First, we transform the initial problem into a graph T-coloring problem. Compared to the initial problem, this new formulation is under constrained. Then we propose a hypergraph modeling to relieve the under constrained problem.

A. Problem 1 - graphs T-coloring problems

Equation (3) indicates that for each SP_j, the sum of the interferences must be lower than a threshold. It means that at least each interference is lower than this threshold. Then we deduce the following binary constraints :

$$(3) \Rightarrow \quad \forall j \in J, \forall i \in I, |x_{i^*} - x_i| \geq t_{ij} \tag{6}$$

where $t_{ij} = \gamma^{-1}\left((\frac{p_{i^*j}}{s_j} - N)/p_{ij}\right)$. t_{ij} is an integer, which can take 6 values, $t_{ij} \in [0; 5]$. (6) are binary constraints similar to those met in the restricted T-coloring problems for graphs.

To illustrate this transformation, let us consider an example of a 3 AP network (AP_1, AP_2 and AP_3) and a user SP_j. The *RSS* received by SP_j are : $p_{1j} = -51dBm$, $p_{2j} = -77dBm$ and $p_{3j} = -75dBm$. Here SP_j is associated to AP_1, the two other AP are jammers. The *SINR* threshold for SP_j is : $s_j = 24dB$ (corresponding to $36Mbps$ nominal throughput). For SP_j, the constraint (3) gives :

$$p_{2j}\gamma(|x_1-x_2|)+p_{3j}\gamma(|x_1-x_3|) \leq \frac{p_{1j}}{s_j}-N = -75dBm \quad (7)$$

$$\Rightarrow \left\{ \begin{array}{l} |x_1 - x_2| \geq 1 \\ |x_1 - x_3| \geq 1 \end{array} \right. \text{ from (6)} \quad (8)$$

Let $G = (V, E)$ be a finite undirected graph where $V = \{AP_1, AP_2, ..., AP_n\}$ the set of vertices and E the set of edges. For each edges (i, i') of G, we define a 5-vector $(w^k_{ii'})_{1\leq k\leq 5}$. The value of $w^k_{ii'}$ indicates the number of users requiring the constraint $C1^k_{ii'} : |x_i - x_{i'}| \geq k$; then : $\forall (i, i') \in I^2, \forall k \in [1; 5]$,

$$w^k_{ii'} = \sum_{t_{ij}=k, i^*=i'} u_j + \sum_{t_{i'j}=k, i^*=i} u_j$$

Let $C1 = \{C1^k_{ii'}/k \in [1; 5], (i, i') \in I^2, i < i'\}$ be the set of constraints of Problem 1. The objective is then to determine $X \in D^n$ which minimizes the function :

$$f_1(X) = \sum_{\substack{(i,i')\in I^2 \\ i<i'}} \sum_{k\in[1;5]} w^k_{ii'}\delta_{C1^k_{ii'}} \quad (9)$$

where $\delta_{C1^k_{ii'}} = \left\{ \begin{array}{ll} 1 & \text{if } C1^k_{ii'} \text{ is unsatisfied (if } |x_i - x_{i'}| < k) \\ 0 & \text{otherwise} \qquad\qquad\quad \text{(if } |x_i - x_{i'}| \geq k) \end{array} \right.$

Figure 2 shows an example of the graph associated to the Problem 1.

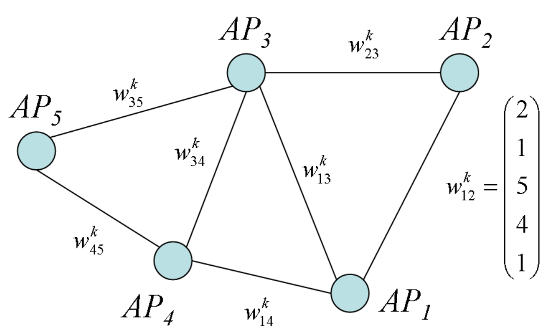

Fig. 2. Example of a graphic repres entation associated to a Problem 1 graph. $w^3_{12} = 5$ means that 5 SP require a 3 channels distance between AP_1 and AP_2.

B. Problem 2 - hypergraphs T-coloring problems

Sometimes, it is possible to have the reciprocal (6)⇒(3). Indeed, in the worst case, we respect the lower limit of the binary constraints of equation (6), i.e. $|x_{i^*} - x_i| = t_{ij}$. If the condition $\sum_{i\neq i^*} \beta^{t_{ij}}_{ij} \leq \tau_j$ is satisfied then (6)⇒(3). We call *binary equivalent SP* the SP which satisfies this condition. We denote $\%e$ the percentage of *binary equivalent SP* of the network.

In other cases, we show that it is necessary to add to (6) an n-ary constraint that has the following form :

$$\forall j \in J, \sum_{i\neq i^*} \alpha_{ij}|x_{i^*} - x_i| \geq \lambda_j \quad (10)$$

where α_{ij} and λ_j judiciously selected.

To illustrate this transformation, we consider the example already presented. Equations (8) are the minimum binary constraints which refer to Problem 1. If we respect the lower limit of these constraints, i.e. $|x_1 - x_2| = 1$ and $|x_1 - x_3| = 1$, we notice that equation (7) is not satisfied : $p_{2j}\gamma(1) + p_{3j}\gamma(1) = -74,3\ dBm > -75\ dBm$. Thus it is necessary to increase either the deviation of the first inequality or of the second one. If $|x_1 - x_2| = 1$, it is necessary at least that $|x_1 - x_3| \geq 1 + 1$ and in a similar way if $|x_1 - x_3| = 1$, it is necessary that at least $|x_1 - x_2| \geq 1 + 2$. In this case, the n-ary constraint to add is :

$$\frac{1}{2}|x_1 - x_2| + |x_1 - x_3| \geq 2, 5 \quad (11)$$

In this example, (7)⇔(8)+(11). Generally, we define :

- t_{ij}, yet defined in the Problem 1, corresponds to the minimal deviation to be respected between vertices i and i^* to satisfy SP_j.
- t^+_{ij} the additional deviation to be added between the vertices i and i^* to satisfy SP_j if the others deviations are minimum ($= t_{ij}$).

Let us consider $\alpha_{ij} = 1/t^+_{ij}$ et $\lambda_j = 1 + \sum_{i\neq i^*} t_{ij}/t^+_{ij}$. In many cases we can show that (3)⇔(6)+(10).

The objective is then to determine $X \in D^n$ which minimizes the function :

$$f_2(X) = f_1(X) + \sum_{j\in J} u_j\delta_{C2_j} \quad (12)$$

where $C2_j : \sum_{i\neq i^*} \alpha_{ij}|x_{i^*} - x_i| \geq \lambda_j$.

Notice that constraints $C2$ are a generalization of constraints $C1$. Then, we define a hypergraph $H = (V, \xi)$ where $V = \{AP_1, AP_2, ..., AP_n\}$ the set of vertices and $\xi = \{E_1, E_2, ..., E_r\}$ a family of V parts. To each hyperedge E_j corresponds either a binary constraint $C1_j$ or a n-ary constraint $C2_j$; For all hyperedges E_j with a dimension higher than 2, there is a principal vertex which corresponds to the AP server. Figure 3 shows a representation of 3 hyperedges : one n-ary and two binaries.

IV. Tests and Results

We carried out WLAN frequency planning tests to compare the three models: Problem 0, Problem 1 and Problem 2. We compare the best results obtained by these 3 approaches. Since problem 0 is the reference modeling, solutions of Problem 1 and 2 are evaluated relative to fitness: f_0.

Testbeds are carried out on multi-floor buildings. The number of AP (n) and SP (m) varies according to the tests. A SP always represents 0,1 user; i.e. $u_j = 0.1$. In the tests, the *SINR* threshold is the same for all users. Before the optimization

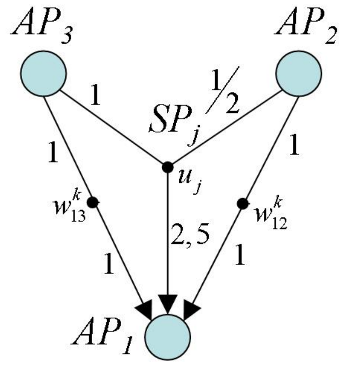

Fig. 3. Graphic representation of 3 hyperedges presented in the numerical example :

$$\begin{array}{llllll} \frac{1}{2} & \times\|x_1 - x_2\| & + & 1 \times \|x_1 - x_3\| & \geq 2,5 & \Rightarrow u_j \\ 1 & \times\|x_1 - x_2\| & & & \geq 1 & \Rightarrow w^k_{12} \\ & & & 1 \times \|x_1 - x_3\| & \geq 1 & \Rightarrow w^k_{13} \end{array}$$

stage, a propagation model computes the *RSS* (p_{ij}) on the whole building. This propagation model is rather realistic : it takes into account the shadowing, reflexion and diffraction effects.

We use the same algorithm to solve the 3 problems, only the fitness changes : f_0, f_1 and f_2. The frequency assignment algorithm we used is a single local search method based on iterative neighbourhood exploration. First, it generates a random frequency plan. For each AP (from 1 to n), we test all the possible frequencies and the best one is affected. The exploration is stopped when no more improvement of the fitness is possible. The algorithm is called *multi-start* because we repeat this process several times (100 or 1000 times). The initial solution is always a new random frequency plan. The algorithm is detailed below :

```
solution =random
For number of start = 1 to 1000
     |∀i, x_i =random between 1 and 13
     |    |x̂ = x
     |    |While the solution is improved
     |    |    |For i = 1 to n
     |    |    |    |For channel = 1 to 13
     |    |    |    |x_i =channel
     |    |    |    |If  (f(x) < f(x̂))
     |    |    |    |    |x̂ = x
     |    |    |    |    |the solution is improved.
     |If  (f(x̂) < f(solution))
     |    |solution = x̂
```

Each test is characterized by its ID, the number of AP, the number of SP, the SINR threshold required by SP and the number of multi-start : "*ID-number_of_AP/number_of_SP/SINR_threshold/ number_of_multi-start*". For example 7-15/3024/16/1000 refers to the 7th test which counts 15 AP, 3024 SP requiring a SINR threshold of 16dBm ($\forall j \in J, s_j = 16$); and the optimization is done with 1000 multi-starts.

The more AP there are, the more important the combinatory is; and the more difficult the problem is . The higher the SINR threshold is, the more unsatisfied the constraints are.

Table 2 summarizes the solutions given by the algorithm. The first column represents the test parameters. The others columns indicate obtained results for Problems 0, 1 and 2. For each problem, we indicate the optimization duration (d) and the fitness f_0 of the best solution. Moreover, for Problem 1, we indicate the percentage of *binary equivalent SP* (denoted $\%e$) and for Problem 2, we indicate the number of additional n-ary constraints ($nb\ c$) we added to Problem 1.

During the tests, the number of AP varies; four cases are studied : 9, 15 , 30 and 40 AP. The *SINR* threshold varies from 4 to 30 ($4dB,\ 16dB,\ 22dB,\ 24dB$ and $30dB$). For each *SINR* threshold corresponds a nominal throughput (cf. table 1).

SINR (dB)	4	16	22	24	30
trougtput (Mbps)	1	11	24	36	54

Table 1. Nominal throughput according to *SINR*. For example, if $4 \leq SINR < 16$ and then $d_c = 1Mbps$

Tests	Problem 0		Problem 1			Problem 2		
ID-$n/m/$*SINR/it.*	d	f_0	d	f_0	$\%e$	d	f_0	$nb\ c$
1-9/3024/4/100	10	0	0.5	0	83	2.5	0	185
2-9/3024/16/1000	187	1.8	9	2.1	43	59	3.2	395
3-9/3024/22/1000	191	20.6	10	21.2	27	54	21.2	332
4-9/3024/24/1000	187	29.2	10	29.2	25	56	29.2	316
5-9/3024/30/1000	197	85.4	10	96.7	19	59	96.7	336
6-15/3024/4/100	35	0	2	0.5	60	13	0	421
7-15/3024/16/1000	689	20	37.7	24.2	26	146	21.8	358
8-15/3024/22/1000	652	70.9	39	79.8	15	136	79.8	321
9-15/3024/24/1000	624	89.8	41	111.6	12	149	111.6	311
10-15/3024/30/1000	616	137	43	160	7	136	158.4	280
11-30/7728/4/100	835	0	37	0.6	85	108	0.4	775
12-30/7728/16/200	2182	35.6	95.6	36.1	58	393	35.1	1383
13-30/7728/22/100	1164	78.5	45	80.4	44	180	73.5	1484
14-30/7728/24/100	3416	101.3	157	103	40	668	104.1	1492
15-30/7728/30/100	1085	178.5	45	186.5	32	188	186.5	1453
16-40/7728/4/100	1392	0	85.7	1.3	85	192	0	712
17-40/7728/16/100	2028	33	146	33.6	54	421	35.6	1444
18-40/7728/22/100	2231	83.7	137	86.2	38	428	83.3	1538
19-40/7728/24/100	2090	107	142	112.3	33	428	112.3	1528
20-40/7728/30/100	2704	201.4	149	205.6	24	415	201.1	1362

Table 2. Tests results : best solutions are indicated in dark gray, less performant ones are indicated in light gray, and worst ones are in white.

A first analysis of the results shows that :

1) In most cases, the best solutions are obtained with problem 0, except for the tests 12,13,18 and 20.
2) Generally, we obtain better results with problem 2 rather than with problem 1, except for the tests 2,14,17.
3) The optimization duration is always faster for Problem 1 and 2 than for Problem 0. Problem 0 is almost 20 times slower than Problem 1 and 5 times slower than Problem 2. Indeed, Problem 0 calculates for each fitness evaluation as many constraints as there are SP (3024 for the ID 9 test). Problem 1 calculates for its fitness $n \times (n-1)/2 \times 5$ constraints where n is the number of AP (525 for the ID 9 test). Problem 2 calculates for its fitness the $n \times (n-1)/2 \times 5$ binary constraints of Problem 1 and a certain number of additional n-ary constraints (311 for the ID 9 test).

However these first results should be moderated. In 7 tests (ID 3, 4, 5, 8, 9, 15 and 19) the solutions of Problem 1 and 2 are identical. It means that sometimes the binary approximation is interesting and is good enough to reflect the interferences phenomenon correctly. If there is a weak AP density in the network like for the first 5 tests, the interferences relate often to only 2 AP.

The n-ary constraints number added to Problem 1 to obtain Problem 2 is relatively independent of the SINR threshold. This number of additional constraints is approximately equal to 300 for the problems with 9 and 15 AP and respectively 1400 for the problems with 30 and 40 AP.

The percentage $\%e$ indicates the SP proportion whose interferences are binary. With an average of 42%, this indicator must be used to reduce the size of the problem. Treating the binary constraints instead of the QoS complete constraints (2), when they are equivalent, will reduce 1/3 of the total optimisation duration relative to Problem 0. We notice that this percentage $\%e$ decreases when the *SINR* threshold increases. Indeed, the higher the *SINR* threshold is, the larger the interfering strength domain is, therefore more and more the jammers impact the interferences. Equivalence between a QoS constraint and binary constraints thus decreases.

V. Conclusion and future work

We considered several real environments in which the number of AP and the number of SP varied. We compared the satisfaction of QoS obtained by the traditional approach of graph *T*-coloring problem (binary interferences) with a new approach called hypergraph *T*-coloring problem (multiple interferences). The obtained results show that this new approach gives better results that the traditional one.

We also noticed that approximately 1/3 of the QoS constraints are equivalent to simple binary constraints. So other network simplification rules can be introduced. In Problem 2, each SP are entirely represented by two *n*-verctors t and t^+ (see III-B) and by the number of users corresponding to it. Concepts of dominance between hyperedges can be defined. Rules of fusion, suppression and addition of hyperedges can then be defined allowing the hypergraph simplification. In future work, the complexity of this problem will be studied more deeply.

It will also be interesting to adapt other algorithms of graphs *T*-coloring [30][31][33] to hypergraphs in order to improve the results given by the algorithm we proposed.

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